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PROBABILISTIC FINITE ELEMENTS WITH DYNAMIC LIMIT BOUNDS; A CASE STUDY: 17th STREET FLOOD WALL, NEW ORLEANS

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ABSTRACT

The probabilistic approach provides a better understanding of failure mechanisms and occurrence probabilities as well as consequences of failure. Besides, main advantages of the probabilistic design in comparison with the deterministic design are: a more careful, more cost effective, and more reliable design of infrastructures. In the present study a new probabilistic approach is applied to the 17th Street Flood Wall, and its probability of failure under hurricane hazard, considering multiple failure mechanisms is assessed. The Monte Carlo (MC) simulation technique coupled with dynamic limit bounds (DLB) integrated with finite element approach is used. The performance of DLB and MC are compared; *the results present that DLB can be efficiently applied in the process of risk and safety evaluation of dikes and flood defenses.*

INTRODUCTION

Flood defenses, notably geotechnical structures, protect people from flooding in vulnerable areas; their failures usually bring a lot of casualties as well as a blow to the economy. The cartoon presented in Figure 1 shows the importance of a typical flood defense is especially in the area below sea level. A recent study shows that roughly one percent of the total population of the flooded area are likely to be casualties of an inundated area (S.N.Jonkman, 2007). The impacts on the economy, however, depend on the flooded area. For instance, if an industrial area is flooded, the economic loss will be considerable. In the other hand, global warming and an increase of normal sea levels bring higher storm surges and increase the risk of flooding of many populated cities. In addition, this phenomenon enhances the importance of flood defenses. As a result, the flood defenses are currently receiving more attention by engineers, societies, and decision makers. Therefore, not only the matter of the design but also the matter of reliability and risk estimation of the available flood defense system is important. In fact, some of the current methods that are being applied in the risk assessment process of flood defenses sometimes provide inaccurate results (M. Rajabalinejad, 2007). Nevertheless, more accurate and detailed models, which are usually modeled with finite elements (FE), can not easily be applied. There are quite a number of studies concerning the improvement of the reliability methods which can be used; still, the probabilistic finite elements (PFE) cannot be applied in the field. This research forms part of the author's PhD research, which tries to fill this gap by presenting a new reliability method suitable for the risk and safety assessment of the flood defense system. Taking the advantage of monotonic behavior of parameters regarding the stability of model, dynamic limit bounds (DLB) are defined and coupled with Monte Carlo (MC). This technique has two main advantages: first, it dramatically reduces the calculation efforts while the accuracy is conserved. Second, the boundaries can be stored and used for the upcoming simulations. Therefore, the whole process of the PFE is described in this paper, which presents a way of accurately estimating the risk and safety of flood defenses. To present the overall process, 17th Street Flood Wall in New Orleans is selected as an important failure of a flood defense system occurred by the Hurricane Katrina. Despite of the design process of the 17th Street Flood Wall,

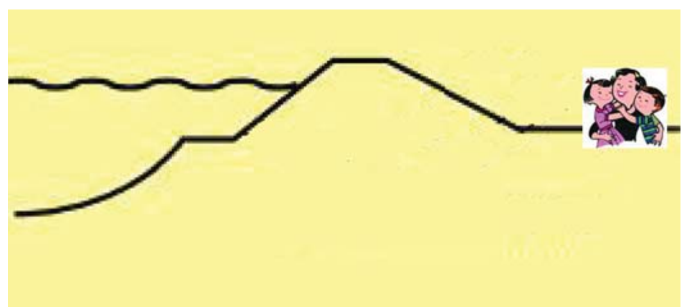

Figure 1. Call attention to the importance of flood defense system primarily in the low land areas.

which is not addressed in this research, the issue of risk assessment of the present flood defenses is discussed in this paper.

FLOOD DEFENSE SYSTEMS

Flood defense systems are different from country to country and place to place depending on the type of floods, available materials, and knowledge of the people. For instance, in Bangladesh the river flood is accepted in the plain area, and it is beneficial; so, people tolerate occasional flooding. However, sometimes it comes bigger than what is expected and makes calamity. In Malaysia the flash flood as a result of heavy rainfall hits populated areas during the monsoon season. In India, also, massive rainfall during the monsoon season expected causing heavy damages. The flood defense, therefore, are some times temporal or permanent in those area.

In some area, nevertheless, people pay more attention to the flood defense system, especially when the area is lower than the normal water level. In other words, there is a permanent risk of flooding in those areas. For instance, the main threat in the Netherlands comes from flooding as is explained in more details in this paper. In the United States, also, floods are expected as a result of rain falls or storm surges and there are some low level lands like New Orleans. Hurricane Katrina brought one of the recent floods caused a lot of casualties and gave the economy of New Orleans a big stroke. The designed flood defense system of this city, in fact, failed to protect the city from flooding.

THE IMPORTANCE OF FLOOD RESEARCHES IN THE NETHERLANDS

There are similarities between the Netherlands and New Orleans from the safety point of view. In fact, the main parts of both lands are below sea level and they are protected by levees, dikes, barriers, and other flood defenses. Besides, the main industrial areas and densely populated area of the Netherlands lie below normal sea level as it is shown in Figure 2. It shows that the capital, Amsterdam, and some of the biggest cities like Rotterdam and The Hague are below sea level. Therefore, a careful management of the flood defenses is vital in the Netherlands as well as doing research about the assessment of the current situation of flood defenses which motivates us to do research about the different aspects of failure of the flood protection system in New Orleans.

THE FLOOD PROTECTION SYSTEM IN NEW ORLEANS

The flood protection system in New Orleans is a combination of levees, flood walls, barriers and some other elements as presented in Figure 3. This system works like a series system meaning that a partial collapse guides to failure of the whole system. Therefore, in flood risk assessment and management it is important to consider all elements when a system's reliability is calculated.

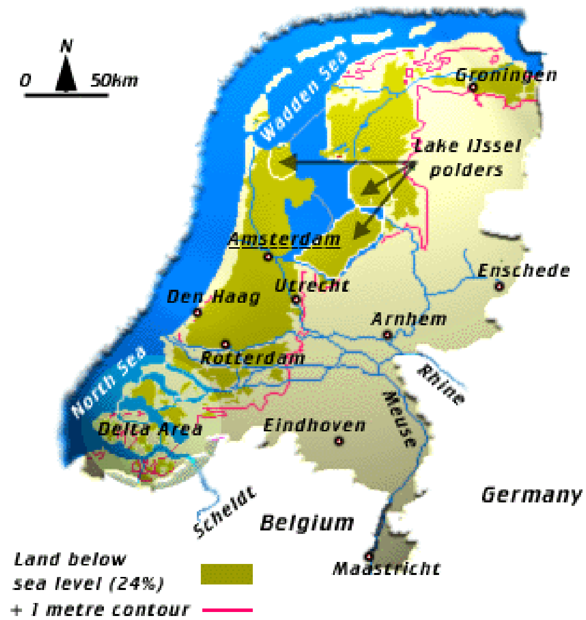

Figure 2. The vulnerable areas in the Netherlands against flooding (Society, 2007).

THE RESILIENT PROTECTION SYSTEM

'Resilience was not an element in the New Orleans Hurricane Protection System design' (USACE, 2006) was one of the lessons which we should learn from that catastrophe. Resilient design can be defined as the ability to withstand, without a complete failure[I] even in the conditions beyond those intended in the design ((USACE), 2006). The resilient design in flood defenses can be defined as follows: a resilient flood defense is a system which doesn't collapse if overtopping or overflowing occurs during the expected period of time.

The concept of *Resilient Design* is not normally applied in engineering design. The resilient design can be accounted as the next step after the reliable design. This concept, especially in the case of flood defenses, is very important and should be clarified for the consultants and designers of flood defenses. In the other hand, a resiliently designed flood defense can provide enormous advantages. For instance, as demonstrated in the analysis of Katrina in New Orleans, the flooding could be reduced to approximately one-third if there was no breach in flood defense system (or if the flood defense system was designed resiliently) (USACE, 2006).

[I] Partial failures are allowed; but the structure should not be collapsed.

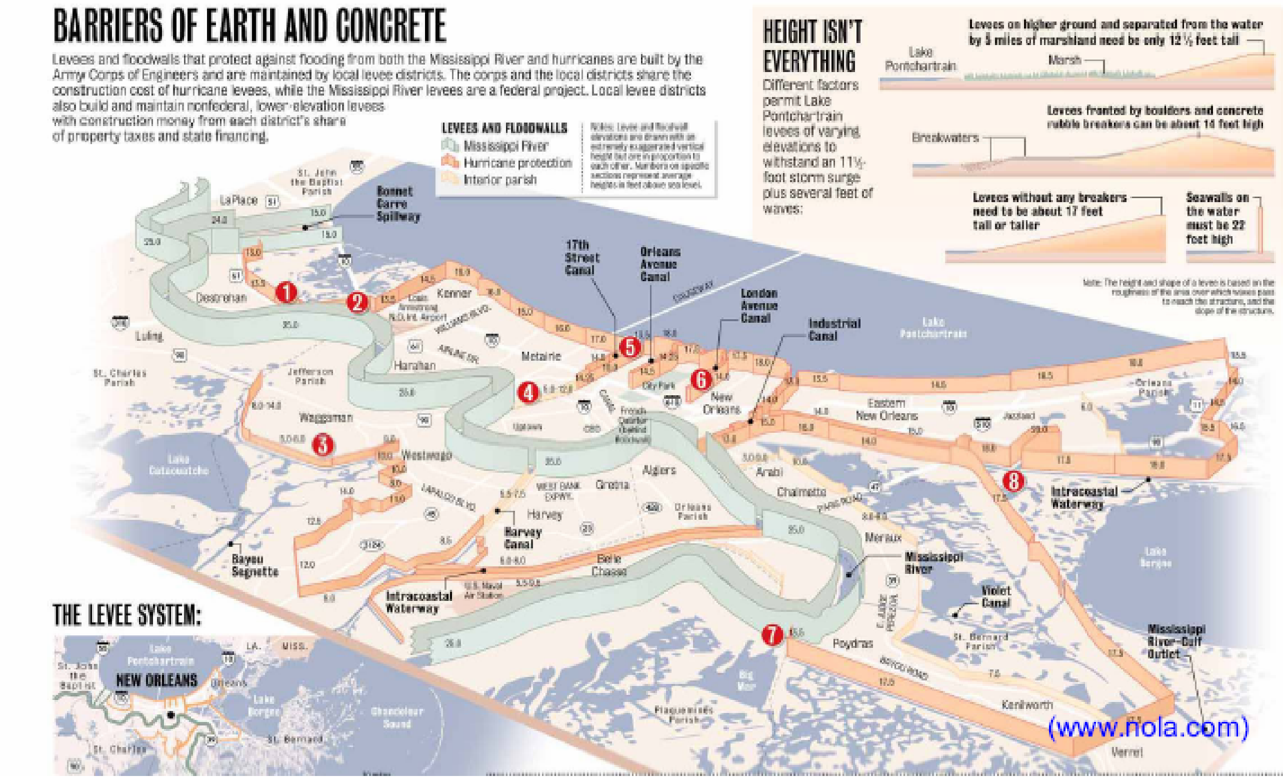

Figure 3. Different elements of flood defense system in New Orleans are presented in this figure [www.nola.com].

AIMS AND OBJECTIVES

Here it is tried to show up a better understanding of the behavior of flood defenses and a broader spectrum of physical behavior based on physical behavior of engineering components, systems, and parameters variation. Moreover, it is shown that the probabilistic approach is a more powerful method enabling us to more accurately model the data and properly understand the contribution of stochastic parameters in a system's failure. In fact, on the suggested base of this paper, existing infrastructures or projects can be reviewed to ensure that their original design has not been compromised by changing hazard, changing knowledge base, or variation of relevant elements and their properties.

FAILURE OF THE FLOOD WALL AT 17th STREET CANAL IN NEW ORLEANS

The failure of the 17th Street Flood Wall in the New Orleans was important because of the fact that its failure was not expected under the applied water level. Therefore, this failure will be discussed during the rest of this paper. The location of the 17th street canal is distinguished in Figure 3 by number 5. Figure 5(a) shows the broken flood wall at the 17th Street Canal in New Orleans. This flood wall was displaced by the flood of Hurricane Katrina at the length of 475 feet. The standing flood wall after displacement in this figure shows that the flood wall was shifted by its foundation. This fact was also concluded by the afterward research projects (see (USACE, 2006),(ILIT, 2006)).

The cross section of the 17th Street Flood Wall and its foundation is presented in Figure 5(b). This figure shows the flood wall including a concrete cap and concrete wall (I wall) located over a sheet pile penetrated into the levee, and soil materials as well as the normal water level in the left hand side at the level of +1 feet. Materials of levee from top to bottom are two layers of clay, a thick layer of peat (March), then a layer of mixed clay and clay laid over a thick sand layer. There is, also, a thin layer of sensitive clay located between March and intermix zone.

The failure of the flood wall at the 17th Street Canal was a typical levee failure in Katrina. But, how predictable it was? In this study it is tried to answer this question by showing up a broader spectrum of possible behavior of the typical I-wall structure. Moreover, it is tried to understand the full performance limits of the flood wall and present new approaches for creating adaptive designs based on physical behavior of engineering components, systems, and parameters variations. US Army Corps of Engineers published the result of their research on the Hurricane Katrina and the related events in 2006 ((USACE), 2006). The Independent Levee Investigation Team (ILIT) also preformed an extensive research project mainly on the Levee's performance and failure (ILIT, 2006). These research projects also were in the attention of TUDelft as a university who leads the flood risk researches in the Netherlands aiming to update the country's levee safety. In this paper, the focus is on the application of improved probabilistic finite elements in which the concept of Dynamic Limit Bounds (DLB) is applied. Therefore, the DLB is applied in the safety assessment of the 17th Street Flood Wall.

THE MODELING AND VALIDATION

The failure of the 17th Street Flood Wall was explored by USACE in an extensive research including a laboratory model and finite elements model. The fifth volume of their report is under the topic of "the performance of levees and flood walls" in which the 17th Street Flood Wall is fully investigate ((USACE), 2006). The 17th Street Flood Wall is also seriously investigated by the Independent Levee Investigation Team (ILIT) in which the results of in-situ geotechnical exploration and finite elements model of 17th Street Flood Wall are presented (ILIT, 2006). Another source of data is a website which covers many technical and geotechnical reports as well as some early design information (https://ipet.wes.army.mil). These enormous amounts of available data are highly valuable for further research on this flood defense system as a part of them is used in this paper.

Figure 5.(a) The plan view of the flood wall at 17th street canal (ILIT, 2006).

Table 1. Information of different soil variables and their variations. The second column shows the soil number according to Figure 7, then the variables of model, soil mode, soil behavior, and distribution type are presented in this table.

Material	Soil Num.	Var	Soil Model	Soil Behavior	CV	Dis.
Brown Clay	1	C	MC	Undrained	0.2	N
Gray Clay	2	C	MC	Undrained	0.2	N
Marsh U. L.	3	C	MC	Undrained	0.3	N
Marsh F.F.	4	C	MC	Undrained	0.3	N
Sen. U. L.	5	C	MC	Undrained	0.3	N
Sen. L. F.	6	C	MC	Undrained	0.3	N
Intermix	7	φ	SSM	Undrained	0.3	N
Gray H.	8	C	MC	Undrained	0.3	N
Gray V.	9	C	MC	Undrained	0.3	N
Sand	10	φ	MC	Drained	0.3	N

Figure 7(a) shows a finite element model of the 17th Street Flood Wall. This model has the same geometry of the model of Independent Levee Investigation Team (ILIT) (ILIT, 2006); the same geometry of ILIT is accepted for our analysis. However, it is tried to reduce the calculation time and make a simpler model by using the less complicated soil models and fewer meshes. The Mohr-Columb and Advanced Soft Soil model are used to model the soil behavior and estimate the

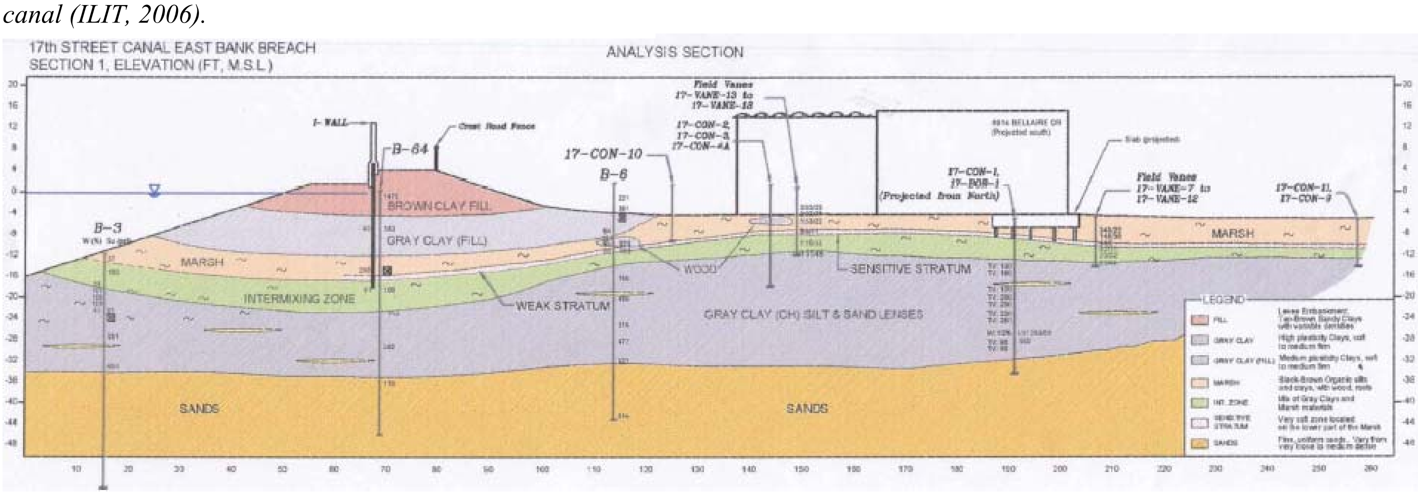

Figure 5.(b) The cross section of the flood wall at 17th street canal (ILIT, 2006).

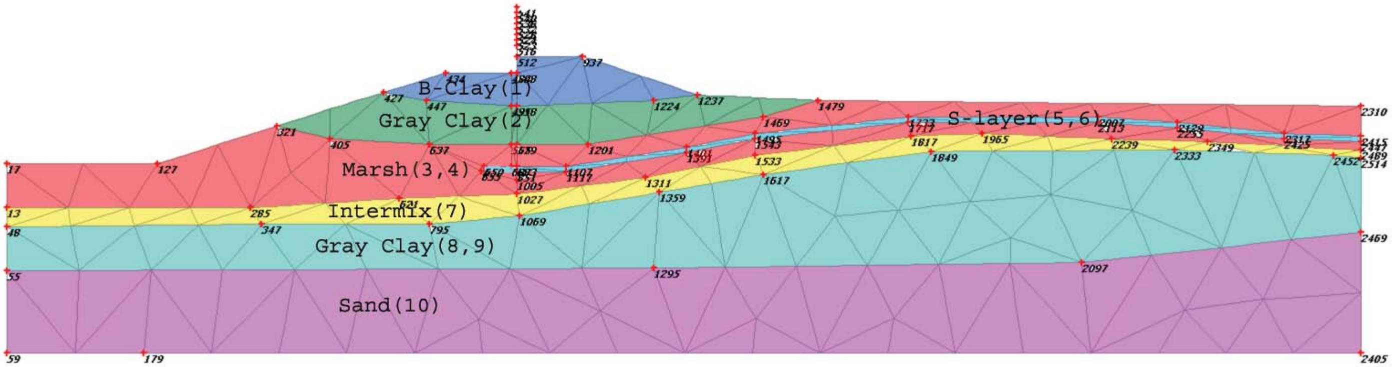

Figure 7.(a) The finite element model of the 17th street flood wall, modeled with Plaxis(Rajabalinejad, 2007b).

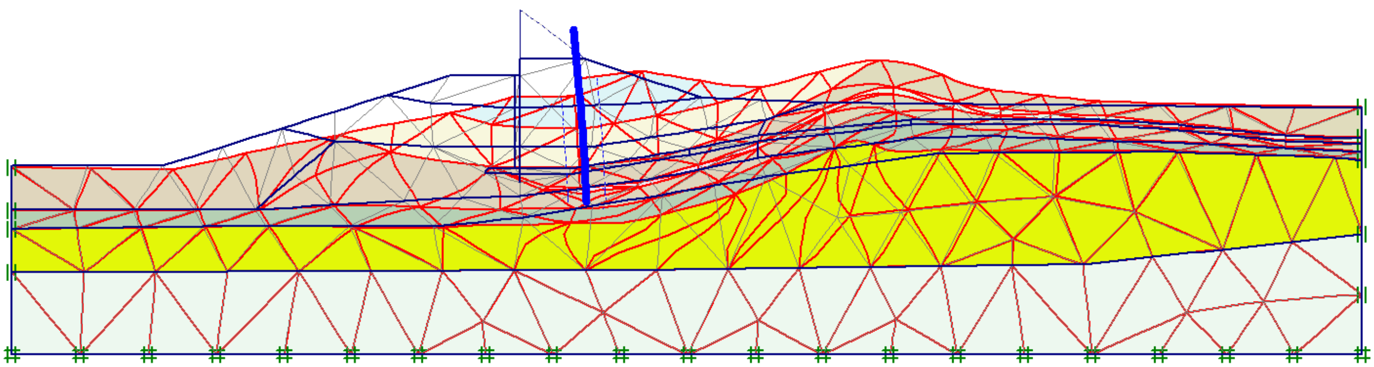

Figure 7.(b) The deformed mesh of model of 17th street flood wall, modeled with Plaxis. The scale factor is 50 (Rajabalinejad, 2007b).

safety considering the fact that the more advanced models are more time consuming; also, Mohr-Columb Model gives satisfactory results for failure prediction. The soil parameters are selected according to the published results of ILIT and USACE; page 8-83 to 8-112 of ILIT (ILIT, 2006) , page V-5 to V-38 of USACE ((USACE), 2006), yet the main reference is ILIT and the results were confirmed by their results.

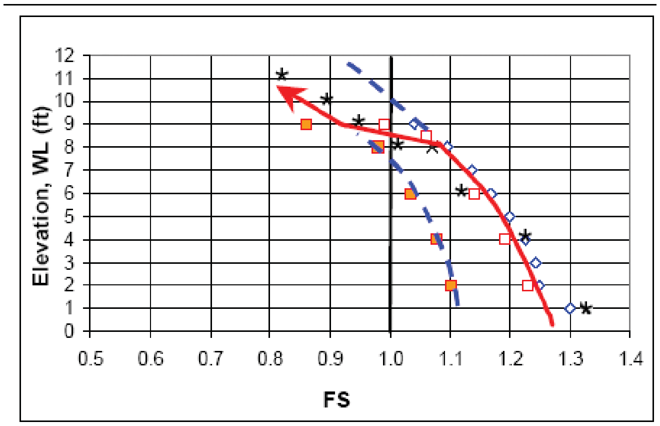

Figure 8. Calculated Safety Factors for three models based on Plaxis analysis of the 17th Street Canal, squares, in comparison with the results of the model used in this research, shown by stars (Rajabalinejad, 2007b).

The 15 nodes triangular elements have been used, and the concrete cap, concrete wall, and the sheet pile are modeled with the linear elastic materials. The interface elements, also, has been used to make the sheet pile impermeable and make a separation between the soil layer and sheet pile wall. This model is used to analyze the behavior of the flood wall MSL +8 feet. The foundation of the flood wall is modeled based on the geometry depicted in Figure 5(b) provided by ILIT (ILIT, 2006).

Figure 8 presents a comparison between the result of the model used in this research by the models of Independent Levee Investigation Team (Rajabalinejad, 2007b). The stars, *, in this figure are the calculated safety factors by mean value of the soil parameters. A well correspondence of the stars and the other (squared) points can be observed in this figure.

LOAD AND RESISTANT VARIABLES

In the probabilistic methods, variables are divided into two main categories of resistance and stress (load). Under this division, a simple form of the limit state function can be defined according to Equation 1 in which there is an implicit or explicit relation between variables and the safety of a model. Accordingly, a limit state equation (LSE) can be defined whereas Z=0.

$$z = Z(\frac{\overline{r}}{\overline{s}}) \qquad (1)$$

LSE: z-*1=0*

Therefore, the limit state equation (LSE) clarifies two different regions where the $LSE \geq 0$ or not. In this case, $\overline{r}$ is a vector of resistant variables, and $\overline{s}$ is a vector of stress variables. The vectors of the probability distribution functions ,PDF, ($\overline{R}$ and $\overline{S}$) subsequently are defined according to Equation 3. For illustration, R_1 is the probability distribution function of r_1 .

However, in complex problems as well as in this research there is an implicit LSE in which the relation between stress and resistant is not explicitly known; furthermore, it is not easy sometimes to make the variables distinguished.

$$\overline{r} = (r_1, r_2, ..., r_p)$$
$$\overline{s} = (s_1, s_2, ..., s_q) \qquad (2)$$

$$\overline{R} = (R_1, R_2, ..., R_p) \qquad (3)$$
$$\overline{S} = (S_1, S_2, ..., S_q)$$

In our model, the water level +8 feet, which was the water level in Hurricane Katrina, is considered as the load. Also, the variations of ten soil parameters are considered in the safety analysis of the 17th Street Flood Wall. Variations are considered both in horizontal and vertical directions according to Table 1.

MONTE CARLO SIMULATION

The Monte Carlo simulation, used in this study, consists of sampling random variables from their statistical distribution and calculating the relative number of simulation for which the limit state is in failure condition (less than zero). Therefore, a relatively large set of random data are contributed in the reliability analysis of the model. As a result, a large set of outputs are produced. Then, the ratio between numbers of failures over total numbers of simulations defines the probability of failure according to Equation 4, where $\hat{P}_f$ is an estimate for the probability of failure. Assuming a large number of simulations, the error (ξ) of $\hat{P}_f$ is assumed normally distributed with the mean value $\mu_\xi = 0$ and standard deviation σ_ξ.

$$\hat{P}_f = \frac{N_f}{N} \qquad (4)$$

$$\xi = \frac{\frac{n_f}{n} - P_f}{P_f} \qquad (5)$$

$$\sigma_\xi = \sqrt{\frac{1-P_f}{nP_f}} \qquad (6)$$

Accepting a 95% confidence interval [II] means that the probability of occurrence should not be smaller than 0.95, this is presented in Equation **7**, and finally Equation 8 gives an estimation of the maximum error.

$$P(\left|\frac{\xi}{\sigma_{\xi_Z}}\right| < 1.96) = 0.95 \qquad (7)$$

In other words, the minimum number of the simulations in Monte Carlo method accepting 95 percent accuracy as follows:

$$n \geq 400 \times (\frac{1}{P_f} - 1) \qquad (8)$$

PROBABILISTIC FINITE ELEMENTS

Probabilistic Finite Element used in this research consists of randomly sampling from the distribution of each input variable and monitoring the behavior of the system under variation of inputs. The Monte Carlo technique is integrated by finite element analysis to provide an accurate estimation of limit state function. For this purpose, a program is written to interactively work with the software package Plaxis: it feeds the Plaxis with the desired probability density functions and gathers the safety factors and correlated variables. This procedure is improved in this research by taking into account the correlation between resistant parameters and outputs. In this case, the dynamic limit bounds (DLB) are applied. The performance of DLB, then, is compared with the Classical Monte Carlo for the 17th Street Flood Wall.

[II] A 95% confidence interval, or 5% percent error is accounted for a good estimation in engineering works.

DYNAMIC LIMIT BOUNDS (DLB)

Monotonic behavior

A function is called monotone with respect to a variable when increasing or decreasing of that variable causes increasing or decreasing of the outputs. In a monotone function, in fact, additional information about its behavior are implicitly applied; for instance, any true system in a logical monotone system will continue to be true by increasing of its variables. Therefore, assuming an n dimensional LSE ($Z(x_1,...,x_n)$), this function can be a monotonically increasing or decreasing function with respect to the variable x_i when Equations 10 or 11 are respectively hold.

$$Z(\overline{x}) = Z(x_1,...,x_n)$$

$$h_i(x) = (x_1, x_2, ..., x_{i-1}, x_i, x_{i+1}, ...x_n) \quad (9)$$

$$x_{m+1} \geq x_m \Rightarrow h_i(x_{m+1}) \geq h_i(x_m) \quad (10)$$

$$x_{m+1} \geq x_m \Rightarrow h_i(x_{m+1}) \leq h_i(x_m) \quad (11)$$

Monotonicity is a normal property of engineering problems and in geotechnical engineering; In other words, knowing the resistant and active parameters, a stable system will remain stable by increasing of the resistant parameters or decreasing of the active variables. For instance, considering a sandy dike which protects the downstream side from, the failure of this dike, therefore, is dependent on the friction angle of soil, φ, as a resistant variable and the water level, h, as the active variable. Then, the stability of this dike is a monotonically increasing function regarding the resistant variable, φ, and monotonically decreasing function regarding the load h.

Thresholds

The threshold concept is widely used in engineering language and determines the difference between levels. This concept divides a set into several subsets with the common desired properties and makes a logical judgment possible to apply. For instance, $F_s = 1$ is a threshold for the factor of safety (F_s) defined as a ratio of resistance over driving forces, F_s =Resistance/Force. The concept of threshold is interesting from the point of view that, if a monotone model is stable and its resistant parameters are increased then the model would remain stable. Furthermore, that model will remain unstable by decreasing of resistant variable.

DLB

Having monotonicity in the limit state function helps us to define two bounds called as upper and lower bounds as a set of respectively upper and lower thresholds ($\{s_{ut}\}$ and $\{u_{lt}\}$), as well as stable and unstable points in Monte Carlo simulation. As a result of these two boundaries, the whole rang of the LSE, $z = (\overline{x})$, is divided into three regions which are the stable region (where $z = (\overline{x}) \geq 0$), the unstable region (where $z = (\overline{x}) < 0$), and the region in between which is called the unqualified part. It is called unqualified because it is a region of the combined safe and failure; it means that in order to get the value of the LSE in this region, unqualified part, we need to evaluate the LSE. This concept is depicted in Figure 9 for a two dimensional joint probability distribution function of variables x_1 and x_2 . This figure, also, presents a schematic view of average distance of limit boundaries from the LSF; it shows that by increasing the number of calculations, the boundaries get closer and the difference between the boundaries get smaller, (See (Rajabalinejad, 2007a)).

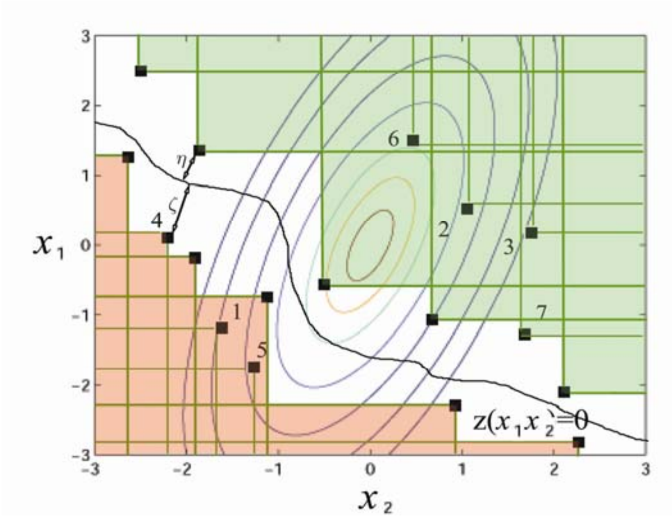

Figure 9. The idea of Dynamic Limit Bounds in a two dimensional space is presented in this figure.

CONTRIBUTION OF SOIL PARAMETERS IN FAILURE

The contribution of every variable, x_i , in the estimated probability of failure can be established according to different tools. We call each variable, x_i , a base variable and z the predicted variable. For instance, in case of the flood wall, the soil parameters are base variables and the calculated safety factor is the predicted variable. Therefore, the correlation between a base variable and the predicted variable determines its contribution into the failure. In simpler terms, a higher correlation between a basic variable, x_i , and the predicted variable, z , a bigger contribution of that variable to the failure is expected.

The rank correlation, presented in Equation 12, is usually used in engineering applications. It is based on the linear correlation between base variable, x_i , and the predicted variable, z. Since, we can not be concern about the linear relation of base and predicted variables, all three methods explained in this section are applied to the flood wall and the results are compared with classical Monte Carlo.

Product moment correlation (ρ)

The product moment correlation or Pearson product moment correlation defines a linear relation between two variables of x_i (base variable) and z (predicted variable) by Equation 12. The product moment correlation can take values in the interval of $[-1 \;\; 1]$; these two boundary limits present a completely linear relation in between when $z = ax_i + b$ where a and b are two constants.

$$\rho = \frac{Cov(x_i, z)}{\sigma_{x_i}\sigma_z} \tag{12}$$
$$Cov(x_i, z) = E[x_i z] - E[x_i]E[z]$$

Correlation ratio *(CR)* and linearity index

Correlation ratio of the base and predicted variables (x_i , *z)* is the square product moment correlation between *z* and function ($f(x_i)$) which maximizes this correlation according to Equation 13. In the other hand, this equation is maximized if $f(x_i) = E[z \mid x_i]$ (Kurowicka D., 2006), therefore:

$$CR(x_i, z) = \max_f \rho^2(x_i, f(x_i)) \tag{13}$$
$$CR(x_i, z) = \rho^2(x_i, E[z \mid x_i]) = \frac{Var[E[z \mid x_i]]}{Var[z]} \tag{14}$$

Equation 14 presents a ratio of the variance of the conditional expectation of z given x_i and the variance of z . Since the squared of product moment correlation is less than or equal of $CR(x, z)$, Equation 15 can measure the linearity of $E[z \mid x]$; therefore, the bigger difference, the higher nonlinear relation is expected.

$$\rho^2(x_i, E[z \mid x_i]) - \rho^2(x_i, z) \tag{15}$$

Rank correlation (ρ_r)

Spearman rank correlation is a good measurement for two variables which are nonlinearly related and they have monotone relationship. As a matter of fact, the rank correlation is a good option which presents the relation between parameters in the problems with monotonic behavior. Spear man rank correlation is defined by the following Equation:

$$\rho_r(x_i, z) = \frac{C_{x_i} + C_z - \sum_{j=1}^{n} d_{ij}^{\,2}}{2\sqrt{C_{x_i} C_z}} \tag{16}$$
$$C_{x_i} = \frac{n^3 - n}{12} \sum_{t_{x_i}} \frac{t_{x_i}^{\,3} - t_{x_i}}{12}$$
$$C_z = \frac{n^3 - n}{12} \sum_{t_Z} \frac{t_Z^{\,3} - t_Z}{12}$$
$$\sum_{j=1}^{n} d_{ij}^{\,2} = \sum_{j=1}^{n} [R(x_{ij}) - R(z_j)]^2$$

Index t_{x_i} and t_z stand for the number of observations of x_i and z with the same rank, $R(x_{ij})$ and $R(z_j)$ stand for the rank ordered x_i and z variables (William H., October 30, 1992).

RESULTS

Classical Monte Carlo

Having the number of failures in simulations, it is possible to estimate the probability of failure by Equation 4. The 95% accuracy is accepted. Therefore, the relative standard error is ($V(\hat{P}_f)$) < 0.05. The estimated probability of failure and number of calculations are presented in Table 2. The result, which is for the water level of +8 feet, presents a high probability of failure. In other words, given the applied soil parameters, soil variation, and the water level, it is highly probable that the flood wall fails to resist. Also, it is assumed in MC simulations that model is stable under its weight itself by any combination of inputs. This condition can be assessed in the zero phase of the Plaxis analysis. Therefore, the randomly generated data which cause instability of model at this phase are not accepted. This assumption means that the variations of input variables are modified[III].

Table 2. The calculated probability of failure by classical Monte Carlo method.

Number of Simulations	W.L.(ft)	$\hat{P}_f$ (%)	N_f	$N \geq$	$V(\hat{P}_f)$
1218	+8	43.6	687	600	<0.05

[III] The variations of soil parameters were assumed to be normal with a usual coefficient of variation (see Table 1). From the previous studies it is concluded, however, that a higher CV, a higher probability of failure is expected [3].

Influence of variables on the failure

The correlation of every basic variable, x_i, with the predicted variable is calculated according to the explained methods. Table 3 presents the calculated product moment correlation (ρ), correlation ratio (CR), and rank correlation (ρ_r). This table is ranked according to the value of product rank correlation for MSL +8 feet.

According to this table, it is clear that the Marsh layer and Gray Clay layer (Layer number 3 and 8 in Figure 7(a)) have the biggest influence factors. This conclusion is certified by the product moment correlation (ρ), correlation ration (CR), and rank correlation (ρ_r). Yet, selection of the third influential variable depends on the ranking criteria. It was discussed that the product moment is suitable when the relation between the basic variables, x_i, and the predicted variable, z, is linear. The correlation ratio also needs interpolation of variables to be calculated as shown in Equation 14. Here, the third order polynomial is assumed for interpolation according to the visualization of data and regression coefficient; however, the value of *CR* is sensitive to the interpolation function. For instance, $E(z\,|\,x_3)^{\mathrm{IV}}$, where x_3 is the soil number 3, is presented by Equation (17).

$$E(z|x_3) = -0.0182+0.0055x_3-0.0000x_3^2+0.0000x_3^3. \quad (17)$$

The rank correlation has two advantages and provides a good criterion for ranking of correlations. First, it can be used in nonlinear relations. Second, it is a suitable choice when there is monotonicity, which is also an essential assumption in DLB, in the model. Therefore, the Spearman ratio or rank correlation sounds to be a good option for ranking the variables in geotechnical flood defense problems. This conclusion is investigated for the 17th Street Flood Wall.

Table 3. The contribution of different soil parameters into the probability of failure regarding different water levels.

Material	Soil Num.	Var	CV	Water level +8 ft		
				ρ(%)	*CR*(%)	ρ_r(%)
Marsh U. L.	3	C	0.3	35.9	16.8	*49.3*
Gray H.	8	C	0.3	24.2	14.9	*54.7*
Gray Clay	2	C	0.2	19.4	10.3	*23.0*
Intermix	7	φ	0.3	15.54	4.2	*12.3*
Gray V.	9	C	0.3	9.3	0.69	*18.3*
Marsh F.F.	4	C	0.3	10.4	0.7	*28.3*
Sen. L. F.	6	C	0.3	2.1	0.05	*4.2*
Sen. U. L.	5	C	0.3	0.7	0.01	*2.5*
Brown Clay	1	C	0.2	-2.3	0.05	*2.1*
Sand	10	φ	0.3	-2.8	0.04	*4.5*

The finite element model introduced in Figure 7 is used in the probabilistic approach and variation of soil parameters are considered according to the Table 1. It also is important to keep in mind that the values of coefficient of variations are assumed as it is normally expected for soil layer; however, the main purpose of this research is showing the robustness of DLB when it is being coupled with Monte Carlo method for a limited number of variables even for a complicated flood defense. This approach, in fact, can be applied in many geotechnical structures and flood defenses. In fact, our aim is providing a more accurate method for estimation of the reliability of flood defenses in the accurate and cheap way.

MONTE CARLO COUPLED WITH DLB

Dynamic Limit Boundaries (DLB) provides two important advantages when it is coupled with Monte Carlo simulations. The first advantage is making the simulation faster when there are a limited number of variables. The second advantage, moreover, is storing the produced limit bounds for the next simulations. These two properties can help practically bringing the probabilistic finite elements from research into the field of risk assessment of flood defenses. Besides, considering the fact that the efficiency of DLB increases by increasing of equivalent Monte Carlo, this method is suitable when there is need for a very high numbers of Monte Carlo simulations (Rajabalinejad, 2007a). Therefore, the probabilistic finite elements can be accurately applied to calculate safety and reliability of flood defenses. However, the efficiency of DLB reduces with increasing of dimensions; yet, in many cases there are only a limited number of influential variables which predominantly determine the probability of failure; the effect of dimensionality is investigated in the next section.

DLB considering first two influential variables

The first two influential variables are selected from Table 3 according to their influence on the stability of the 17th Street Flood Wall. These variables are presented in Table 4. In fact, all of the ranking methods which are described in this paper guide to this selection. Therefore, these two variables are the main influential soil layers playing the main rule in failure of the structure. This result also was concluded in the previous research (Rajabalinejad, 2007b).

Table 4. The contribution of different soil parameters into the probability of failure.

Material	Soil Num.	Var.	Soil model	CV	ρ(%)	ρ_r(%)
Marsh L.	3	C	MC	*0.30*	35.9	49.3
Gray C. H	8	C	MC	*0.30*	24.2	54.7

Table 5. Results of reliability analysis considering variation of the first two influential variables.

W.L. (feet)	DLB	failures	Stable	Equivalent MC	P_f (%)	V (P_f)
+8	119	603	*897*	1500	40.2	*<0.05*

Table 5 presents the results of DLB method considering the first couple of influential variables. The second column of this table shows the calculated number of DLB. As a result, a narrow confidence interval for $\hat{P}_f$ is obtained.

DLB considering first three influential variables

The three dimensional DLB is coupled with the Monte Carlo to estimate the safety of the 17th Street Flood Wall. In this case, the first tree influential variables are to be considered. In the other hand, different ranking criteria give different output. In fact, *a good criterion is essential for ranking variables in DLB technique*.

To recognize the best ranking criterion, the results can be compared with the Monte Carlo simulations. Therefore, the closer result to the Monte Carlo simulations, a better criterion is selected. In other words, the influence of variables according to product moment correlation and rank correlation are compared to clarify that which criterion is more efficient for ranking in this case study. Therefore, the first three influential variables are ranked according to the product moment correlation, ρ, and rank correlation, ρ_r, in Table 6 and Table 8, respectively. The correlation ratio, CR, is not considered because its result is so dependent on the interpolation function (Equation 17); a higher degree of interpolation function doesn't necessarily gives a better interpolation function. Table 6 shows that soils with number 3, 8, and 2 are the first, second, and third influential variable in failure of the flood wall for different water levels. These variables are selected according to product moment correlation

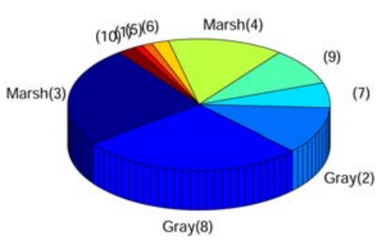

Figure 10. This figure shows the contribution of different variables (see Table 1) into the failure of the 17th street flood wall, New Orleans. The variables are ranked according to rank correlation (ρ_r).

(ρ). Table 8, however, presents that soils with number 3, 8, and 4 are the main influential variables according to the rank correlation (ρ_r).

In the next step, three dimensional DLB is applied to the variables presented in Table 6 and Table 8; the results are accordingly presented in Table 7 and Table 9. A comparison between results of these tables with results of Classical Monte-

Table 6. The contribution of different soil parameters into the failure ranked by ρ.

Material	Soil Num.	Var.	Soil model	CV	ρ(%)	ρ_r(%)
Marsh L.	3	*C*	MC	*0.30*	35.9	49.3
Gray Cl. H	8	*C*	MC	*0.30*	24.2	54.7
Gray Cl.	2	*C*	MC	*0.20*	19.4	23.0

Table 7. Results of stability analysis considering variation of the first three influential variables according to ρ.

W.L. (feet)	DLB	failures	Stable	Equivalent MC	$\hat{P}_f$ (%)	$V(\hat{P}_f)$
+8	221	781	*719*	1500	52	*<0.05*

Table 8. The contribution of different soil parameters into failure ranked by ρ_r.

Material	Soil Num.	Var.	Soil model	CV	ρ(%)	ρ_r(%
Gray Cl. H	8	*C*	MC	*0.30*	24.2	54.7
Marsh L.	3	*C*	MC	*0.30*	35.9	49.3
Marsh F.F.	4	*C*	MC	*0.30*	10.4	28.3

Table 9. Results of stability analysis considering variation of the first three influential variables according to ρ_r.

W.L. (feet)	DLB	failures	Stable	Equivalent MC	$\hat{P}_f$ (%)	$V(\hat{P}_f)$
+8	202	322	*438*	1500	42.9	*<0.05*

Carlo (Table 2) shows that the rank correlation provides better estimation. In other words, *rank correlation does a better job in ranking of the variables according to their correlation with the predicted variable in this case study*. This is an important point for applying DLB.

CONCLUSIONS

In the present study, it is tried to improve the probabilistic method integrated with finite elements analysis by dynamic limit bounds. Considering the monotonic behavior of the model and a limited number of input variables, the performance of this method is compared by Classical Monte Carlo in a complex model of the 17th Street Flood Wall. The results show a good correspondence and accuracy even in three dimensions. Nevertheless, a higher dimension of DLB can still be applied.

DLB method has a memory. The generated bounds can be stored for upcoming simulations; therefore, accurate results are easily accessible for the next series of simulations.

DLB technique is suggested for safety assessment of the available flood defenses. These structures need a more accurate reliability analysis in comparison with what is being applied (M. Rajabalinejad, 2007). DLB makes the probabilistic finite elements cheap and available and PFE can be applied into the flood defenses.

In a monotone function, the rank correlation (ρ_r) seems to be a more accurate criteria for ranking the influence of variables over the probability of failure in comparison with the product moment correlation; this conclusion is verified for a complex geotechnical model with monotonic behavior which is described in this paper.

RECOMMENDATIONS

This research provides a flexible approach for safety assessment of flood defenses; it is highly recommended using this technique to estimate the safety of flood defenses especially dikes and levees.

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LIST OF SYMBOLS

x_i	A random variable
$\bar{x} = (x_1, ..., x_n)$	A vector of random variables
X_i	The Distribution of x_i
$\bar{X} = (X_1, ..., X_n)$	A vector of distributions (PDF)
x_{ij}	A realization from x_i
r_i	A resistant random variable
$\bar{r}$	A vector of resistant random variables
R_i	The Distribution of resistant variable r
$\bar{R} = (R_1, ..., R_n)$	A vector of distributions of resistant variables (PDF)
s_i	A stress (load) random variable
$\bar{s} = (s_1, ..., s_n)$	A vector of stress random variables
S_i	The Distribution of stress variable r
$\bar{S} = (S_1, ..., S_n)$	A vector of distributions of stress variables (PDF)
z	The random variable of limit state equation
z_i	A realization of the z
Z	The distribution of the z
ρ	Product moment correlation (Pearson correlation)
CR	Correlation Ration
ρ_r	Rank correlation (Spearman correlation)
LSF	Limit state function
LSE	Limit state equation
P_f	Probability of failure
$\hat{P}_f$	Estimated probability of failure
ξ	Error of estimation
μ_x	Mean value of variable x
σ_x	Standard deviation of variable x
$E[x]$	Expected value of the distribution of x
s_{ut}	Set of upper threshold points
s_{lt}	Set of lower threshold points

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